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BED DYNAMICS DURING DRUM GRANULATION

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Results of investigations of a granulated bed dynamics during agglomeration in rotating drums were discussed. The investigations were carried out in granulators with continuous wetting of the bed with water, thus making no difference whether the wetting time or bed moisture content was assumed as an independent variable. The variable parameters were: drum diameter D ranging from 0.25 to 0.4 m, drum filling with raw material k in the range from 5 to 20% of inner drum volume and rotational speed of the drum n = 10 to 32 rpm. During granulation the torque value was measured on the granulator shaft. Every minute a sample was taken from the drum, the angle of natural repose of the product, and its bulk density were determined. A dimensionless equation describing the bed dynamics during granulation process was proposed.

Key words: drum granulation, torque, power number, Froude number

INTRODUCTION

The problems of granular bed dynamics were discussed in the literature most thoroughly for ball mills. Much fewer are the studies on mixing, granulation and other unit operations. Dirge (1990) claimed that the value of torque generated by forces of gravity of the bed in the ball mill depends on mill dimensions and properties of the bed. To describe this phenomenon Rose and Sullivan (1958) used dimensional analysis that included many parameters, while Gao (1990) proposed a formula which allowed him to calculate the power demand. Assuming that balls filling the drum are a continuous medium, which circulates in cascades, after a critical value of the angle of inclination has been reached on the free surface, Hogg and Fuerstenau (1972) calculated the work required to lift the balls that moved along circular trajectories from the lower to upper equilibrium point. As a result, they found that power required to drive the drum depended on the drum dimensions, rotational speed, filling with material and on the mass of this filling. Arbiter and Harris (1982) obtained similar results by equilibrating the moment of peripheral (driving) force with the moment of

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braking force of the load. On the basis of experiments, Kapur (1992) obtained the dependence of torque (treated as an energy parameter) on time. A characteristic property of this dependence published in a graphical form, is that the torque is constant in some range, it does not change with time and proceeding granulation. The range of constant torque was also confirmed by the investigations of mixing and granulation carried out by Heim et al. (1999). Weidenbaum (1958) introduced the Froude number to his relations as a parameter characterising the bed dynamics. The Froude number was used also in the description of material motion during granulation by Gusiev (1966) and Korotich (1966), who gave it as a function of a subsequent dynamic parameter – the angle of natural repose. A detailed analysis of the bed dynamics was proposed by Kantorowicz (1959) who studied the distribution of forces acting on granules. After making many simplifying assumptions he found the angle of bed inclination was equal to the angle of natural repose. When analysing the action of forces on grains of the bed and their velocities (both for granulation and mixing) Heim et al. used in the description of dynamics such parameters as bed inclination angle β (1995), moment on the drum shaft (1994), supplied power (1995) and grain velocity (1997).

AIM OF THE STUDY

The aim of the study was to describe bed dynamics during granulation using dimensionless numbers, and to determine the effect of process and equipment parameters (granulator drum diameter, its filling with raw material, rotational speed of the granulator and granulation time), and parameters characterising the granulated product (bulk density, friction angle) on the absolute torque value.

SCOPE OF INVESTIGATIONS

Experiments were carried out in a laboratory rig for drum granulation for the following range of variable parameters:

- granulator drum diameter D = 0.25 to 0.40 m,
- drum filling with a granular bed k = 5% to 20%,
- the ratio of rotational speed of the drum to critical velocity n/n_{kr} = 0.15 to 0.375.

The tested material was foundry bentonite with grain size ranging up to 0.16 mm and bulk density ρ_n = 865 kg/m^3.

MODEL FORMULATION

During formulation of the model the following assumptions were made:

- the bed of granular material is a rigid body,
- the circulation of granular material in rotating drum is caused by the moment M_t generated by friction forces.

Considering the bed dynamics for an elementary sector of surface area dF , we can determine the value of the moment M_t. Figure 1 shows the distribution of forces acting on the granulated bed and its elementary sector. The following forces act upon element ABED.

– friction forces of the bed lifting – dT,
– gravity force – dP = ρ_ngLdF,
– centrifugal force dB = ω^2RdP/g.

The friction force is connected with the centrifugal force and the radial component of the gravity force. It is determined by the formula:

$$dT = f \cdot dP\left[\frac{\omega^2 \cdot R}{g} + \cos(\beta_0 + \varphi)\right] = f\rho_n LgdF\left[\frac{\omega^2 R}{g} + \cos(\beta_0 + \varphi)\right] \tag{1}$$

According to Fig. 1, the elementary surface dF is

$$dF = R^2 \cos\varphi(\cos\varphi - \cos\alpha)d\varphi \tag{2}$$

Taking eqs. (1) to (2) the moment of friction force distributed along the arc on which material is in contact with the drum wall, is determined by the integral

$$M_t = \int_{-\alpha}^{+\alpha} RdT = R^3 Lf\rho_n g \int_{-\alpha}^{+\alpha}\left[\frac{\omega^2 R}{g} + \cos(\beta_0 + \varphi)\right]\cdot\left(\cos^2\varphi - \cos\alpha\cos\varphi\right)d\varphi \tag{3}$$

Upon integration we have:

$$M_t = fgLR^3\rho_n\left[\frac{\omega^2 R}{g}\left(\alpha - \frac{\sin 2\alpha}{2}\right) + \cos\beta_0\left(\frac{4}{3}\sin\alpha - \frac{1}{6}\sin 2\alpha\cos\alpha - \alpha\cos\alpha\right)\right] \tag{4}$$

Because the central angle α is a parameter which defines the drum filling with granular material k (and bulk density ρ_n, and angle β_0 is proportional to the inner friction coefficient f, the obtained formula can be simplified to the form

$$M_t = ALD^3\rho_n^c\left[\frac{\omega^2 D}{2g}\cdot k^a\right]\cdot tg\beta^b \tag{5}$$

where A is constant.
Because power is defined by the formula

$$N = M_t \cdot \omega \tag{6}$$

After dividing both sides of the equation (5) by $\omega^3\rho_n D^5$ and taking into account the definition of dimensionless numbers, we obtain the following relation

$$\frac{N}{\omega^3 \cdot \rho \cdot D^5} = A \cdot \frac{g}{\omega^2 D} \cdot \frac{L}{D} \cdot k^a \cdot tg\beta^b \cdot \rho_n^{c-1} \tag{7}$$

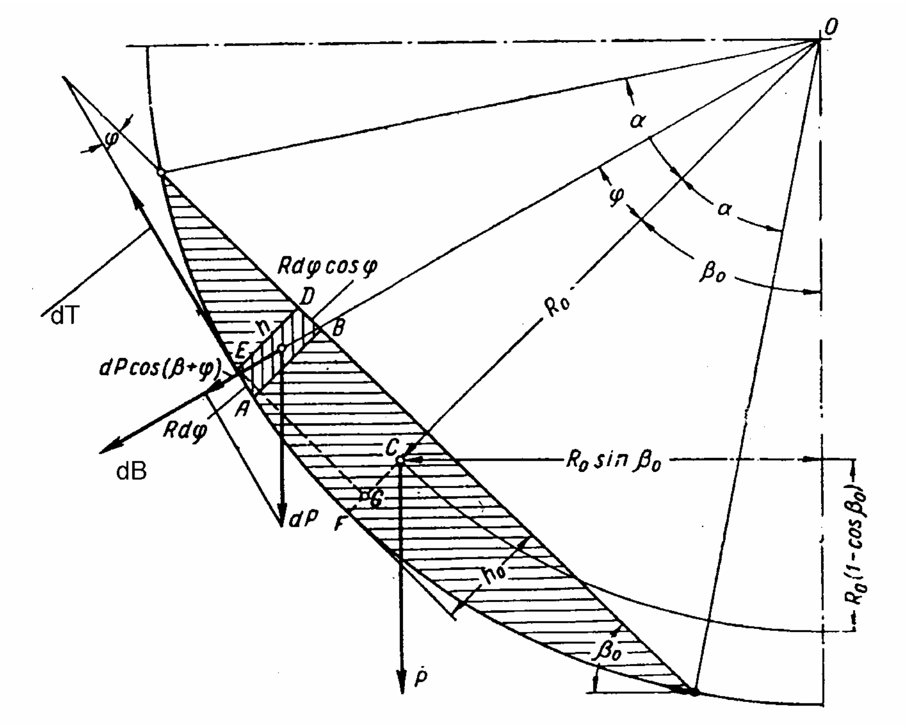

Fig. 1. Forces acting upon the elementary sector of the granulated bed

Upon substituting the symbols of dimensionless numbers: power number (N_e) and Froude number (F_r) we obtain

$$N_e = A \cdot \frac{1}{F_r} \cdot \frac{L}{D} \cdot k^a \cdot tg\beta^b \cdot \rho_n^{c-1} \tag{8}$$

MEASURING EQUIPMENT AND METHODS

Experiments were carried out using an experimental rig for drum granulation. Drums of length L= 0.24 m, were driven by motoreducer by means of a belt transmission and a coupling. A smooth change of the rotational speed of the drum was obtained by means of inverter, and was controlled by a revolution counter.

Instantaneous values of the torque were measured by torque meter, read using reader and processed and recorded by computer. The granular bed placed in the drum was wetted drop-wise by sprinkler inserted axially into the granulator. The sprinkler was mounted on a separate stand.

The wetting liquid was supplied from tank, located 2.5 m above the drum axis and its flow rate was controlled by rotameter. During the experiment the liquid in the tank was maintained at a constant level, which guaranteed a constant flow rate of the liquid Q = 10^{-6} m^3/s. The wetting liquid was distilled water, and the sprinkler ensured its uniform distribution along the entire drum length. The granular bed was wetted at a constant rate of the liquid discharge, until overwetting of the material which caused that the bed got stuck to the inner wall of the granulator. The process was carried out batch-wise, each time at steady process and equipment parameters, i.e. drum filling, rotational speed of the granulator and drum diameter. In the experiments drums of diameters 0.25, 0.3, 0.35 and 0.4 m were used. To be able to compare the results obtained for different drum diameters, we used in the experiments the values of relative velocity n_w, i.e. the rotational speed n related to the drum critical velocity n_{kr}. The relative velocity n_w was changed in the range which ensured an avalanche character of motion of the bed tumbling in the drum. Samples were taken in 60 s time intervals. On this basis bulk density of the bed and the angle of natural repose were determined. Prior to each experimental trial the torque was measured for the idle run of the apparatus.

RESULTS

The reduced moment M*, described by the following formula, was proposed to define the processed bed dynamics

$$M^* = \frac{M - Mj}{m_s + m_w} \tag{9}$$

Examples of changes in the reduced moment M* during granulation (wetting), i.e. with a change of bed moisture content are shown in Fig. 2. The points were obtained by averaging 60 subsequent readings of the torque made every 1 s.

Analysis of the relation M* = f(w) resulted in the determination of three ranges of the moment. The first one for the value growing approximately with the square function, until reaching a maximum, the second one – a short-term decrease called a transient range, and the third one, when the values of reduced moment are constant. To describe the effect of parameters on changes in the absolute values of torque ($M-M_j$) used for bed circulation during granulation, it was decided that – because of a different character of the relations in particular ranges – correlations should be prepared separately for two ranges of the torque change:

– for the first one in which there is an increase of the reduced moment M*, (increase range),
– for the range in which the torque values are constant (stabilisation range).

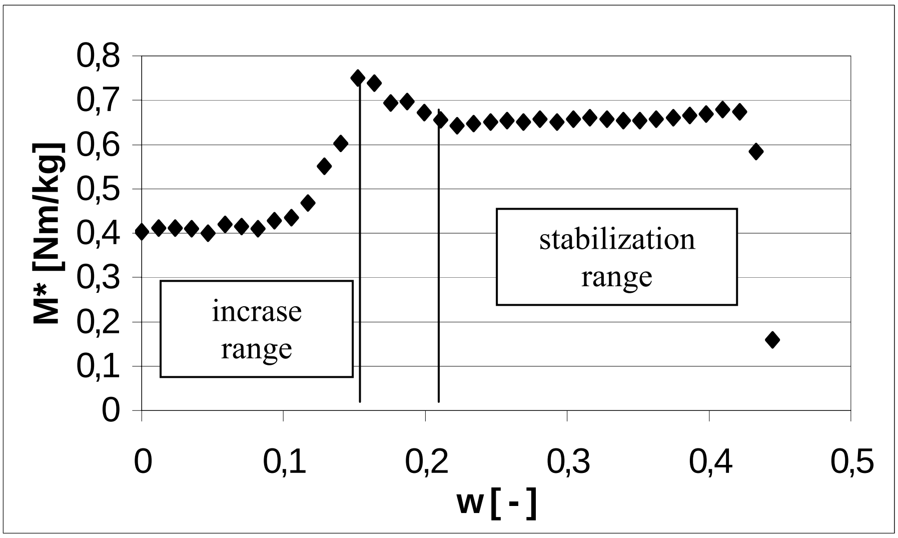

Fig. 2. Examples of changes in the reduced moment value with changing bed moisture content

Two correlation equations were obtained to calculate the torque value and analogously, two dimensionless equations describing the bed dynamics for the two mentioned stages of the process.

The effect of the tested parameters on the torque value was described by the equations:

– for the stabilisation range

$$M_{st} = 100 \cdot D^{2.87} \cdot n^{0.15} \cdot k^{0.75} \cdot \rho_n^{-0.5} \tag{10}$$

at correlation coefficient $R^2 = 0.96$

– for the increase range

$$M_{kw} = 10^{1.44} \cdot D^{2.13} \cdot n^{-0.1} \cdot k^{0.5} \cdot \rho_n^{-2.13} \cdot tg\beta^{1.21} \tag{11}$$

at correlation coefficient $R^2 = 0.905$.

Upon transformation, formulae (10) and (11) can be used to calculate power demand for granulation if the final product density is known.

Analysis of the above relations shows that in the "increase range" the torque value is affected strongly by the parameters that characterise the produced granulated material. Properties of the granular material change rapidly, so the bed tumbles irregularly which has an influence on fluctuations and an abrupt increment of the torque value. For the "stabilization range" the equipment and process parameters have a stronger effect on the bed dynamics.

Additionally, on the basis of the results obtained the values of dimensionless numbers Ne and Fr in the previously described model were calculated. This allowed us to generate the following dimensionless equations.

– For the "stabilization range"

$$Ne = \frac{100}{E_1} = f(Fr) = \frac{100}{Fr} \cdot \frac{1}{\left(\frac{D}{L}\right)^{1.2} \cdot \rho_w^{1.5} \cdot k^{-0.75}} \tag{12}$$

where relative density $\rho_w = \rho/\rho_p$

A dimensionless relation was obtained at $R^2 = 0.98$.

For the "increase range" the relation in a similar form was obtained:

$$Ne = \frac{53}{E_2} = \frac{53}{Fr} \cdot \frac{1}{\left(\frac{D}{L}\right)^{1.8} \cdot \rho_w^{3.2} \cdot tg\beta^{-1.2} \cdot k^{-0.55}} \tag{13}$$

at correlation coefficient R^2 =0.94.

The above formulae of functions confirm the character of relationships between dimensionless numbers proposed in the dynamics model. The relations Ne=f(Fr) for both considered ranges are presented graphically in Fig.3 and Fig. 4.

The dimensionless equations describing the process have the same character of relationship and a similar range of changes in the Euler and Froude numbers for both ranges considered. They differ only in the slopes of the straight lines, which depend on the exponents of dimensionless simplexes ($D/L, k, \rho_w$). This is a result of a different influence the particular parameters have on both granulation ranges.

For the increase range, which occurs at the beginning of the granulation when the bed movement is still unsteady, there is a much more distinct effect of the parameters that characterise the granulated bed. The value of the correlation coefficient and the scatter of points from the mean value confirm that the operation of the apparatus in this range is less stable. This is induced by an uneven tumbling of the granulated bed, which after being lifted to some height detaches and falls down suddenly in a way similar to a slope creep.

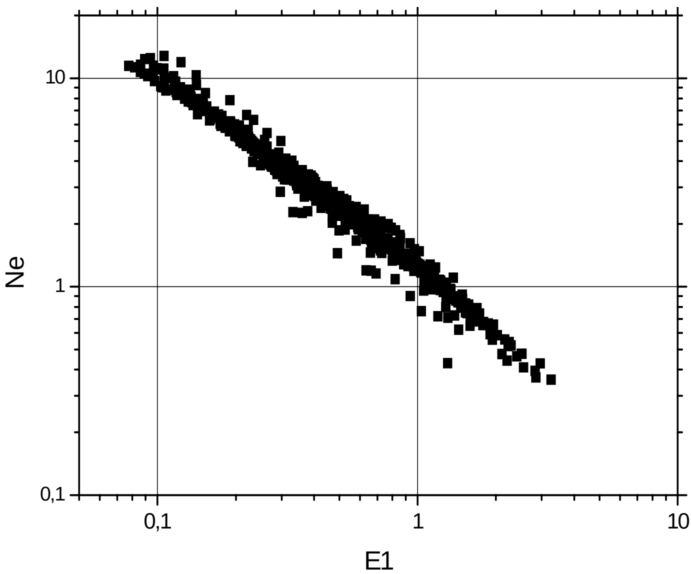

Fig. 3. Dependences of the power number on the value of E_1 describing the effect of Froude number on process dynamics for the stabilization range

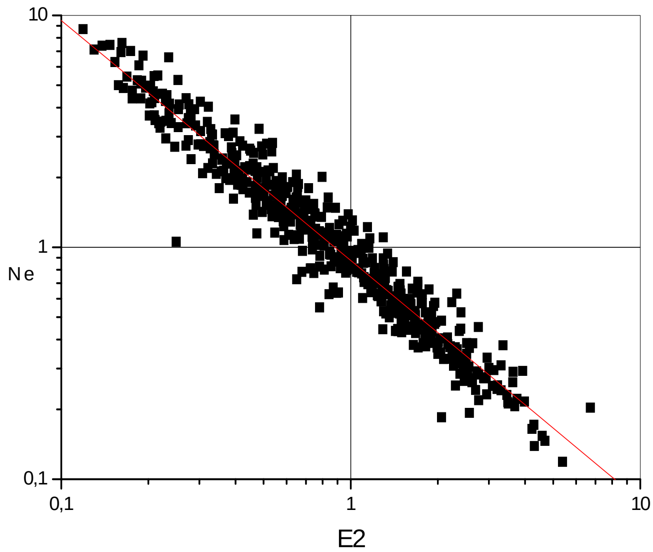

Fig. 4. Dependences of the power number on the value of E_2 describing the effect of Froude number on process dynamics for the increase range

In the case of the stabilisation range a much lesser scatter of the values from the centre line can be observed which gives evidence of a smooth operation and a stronger influence of parameters connected with the values that describe the process conditions.

CONCLUSIONS

1. The proposed model of the process and experimental results show that the bed dynamics during drum granulation can be described by the dimensionless equations presenting relationships between power number, Froude number and dimensionless parameters characterising the granulated product, and also process and equipment parameters. The increment of power number is inversely proportional to changes in the Froude number for all stages of the process.
2. Analysis of changes of the reduced moment M* (parameters of the dynamic process) shows that it increases on the initial stage of granulation and then it assumes constant values. Changes of the granulated product properties at this stage of the process do not affect significantly the behaviour of the entire bed.
3. Correlations in which the torque depends on the operating parameters and granulated product properties were obtained. They are useful in designing the energy aspect of the apparatus.

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NOMENCLATURE

m_s – mass of loose material placed in the drum [kg]
$m_w = Qt_n\rho_n$ – mass of the wetting liquid added [kg]
M – moment on the granulator shaft [Nm]
Mj – moment of idle run for [Nm]
M* – moment reduced on the granulator shaft [Nm/kg]
N – power supply [W]
t_n – wetting time [s]
Q – volumetric flow rate of wetting liquid [m^3/s]
ρ – density [kg/m^3]
$\rho_w=\rho/\rho_p$ – relative density [kg/m^3]
ρ_p – powder density [kg/m^3]
β – angle of natural batching
ω – angular velocity of the apparatus [s^{-1}]
n – rotational speed of the apparatus [rpm]
w – moisture content
A – constant
Ne – power number
Fr – Froude number
$E_{1,2}$ – parameters describing bed dynamics during granulation

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W pracy przedstawiono wyniki badań dotyczących dynamiki granulowanego złoża podczas prowadzenia procesu aglomeracji w bębnach obrotowych. Badania prowadzono w granulatorach przy ciągłym nawilżaniu złoża wodą, co powodowało, że przyjęcie jako zmiennej niezależnej czasu nawilżania bądź wilgotności złoża było tożsame. Jako parametry zmienne stosowano: średnicę bębna D od 0.25 do 0.4 m, stopień wypełnienia bębna surowcem k w zakresie 5-20% objętości wewnętrznej bębna, prędkość obrotową bębna w zakresie n=10-32 obr/min. Podczas granulacji mierzono wartości momentu obrotowego na wale aparatu. W stałych odstępach czasowych równych 1 min. pobierano z bębna próbki, na podstawie których określano kąt naturalnego usypu uzyskanego produktu, a także jego gęstość nasypową. W pracy zaproponowano równanie kryterialne opisujące dynamikę złoża w trakcie procesu granulacji.