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THE DEPENDENCE OF DISTRIBUTION OF SETTLING VELOCITY OF SPHERICAL PARTICLES ON THE DISTRIBUTION OF PARTICLE SIZES AND DENSITIES

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Settling velocity is an independent variable of the hydraulic separation performed for instance by means of jigs. Therefore, the settling velocity characterizes material forwarded to the separation process.

The paper presents a method of determining the distribution of settling velocity in the sample of spherical particles for the turbulent particle motion in which the settling velocity is expressed by the Newton formula. Because it depends on density and size of particle which are random variables of certain distributions, the settling velocity is a random variable. Applying theorems of probability, calculations concerning the functions of random variables, formulas for the frequency function of settling velocity and the distribution of velocities for several combinations of distributions of particle sizes and densities were presented.

Key words: settling velocity, distribution of settling velocity, random variables, function of random variables

INTRODUCTION

Terminal velocity of particle is the settling velocity in the uniform motion, when the geometrical sum of all forces acting upon the particle is equal to zero. There are many methods of determining terminal velocity (Sztaba,1992). In this paper a theoretical method resulting from the solution of the particle motion equation will be used. In the turbulent motion, the force of resistance is expressed by Newton's formula. Therefore the force balance equation and the terminal settling velocity are as follows (Finkey,1924, Sztaba, 2004):

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$$P_{\Psi} = P_N = \frac{\pi}{12}\rho_0 v^2 d^2 = \frac{\pi d^3}{6}(\rho - \rho_0)g \tag{1}$$

$$v = \sqrt{2g}\sqrt{d\frac{\rho - \rho_0}{\rho_0}} = K\, x^{1/2} d^{1/2} \tag{2}$$

where: $K = \sqrt{2g} = 4{,}43\left[m^{-1/2}s^{-1}\right]$ - constant, $x = \dfrac{\rho - \rho_0}{\rho_0}$ - denotes the reduced relative density, d – particle size, g – acceleration due to gravity, ρ - particle density, ρ_o – liquid density, v – terminal settling velocity of a spherical particle.

It results from Eq.(2) the settling velocity of the spherical particle is a function of particle size, its density and properties of the medium in which the particle motion takes place (ρ_o, μ). During separation of heterogeneous materials (from the point of view of their physical and geometrical properties – such as enrichment of coal and ores), both the particle size and its density are random variables of fixed distributions. As a result the particle settling velocity will be a random variable being a function of random variables such as particle density and size. The form of distribution of this random variable results from Eq.(2) and distributions of particle size and density. This paper presents methods of calculating spherical particle settling distribution according to Newton's formula because separation in classifying devices including jig takes place in a turbulent motion, for which the particle settling velocity is calculated from Eq.(2).

THE DISTRIBUTION OF SETTLING VELOCITY ACCORDING TO NEWTON'S FORMULA

The reduced relative density of particle x occurs in the formulas of particle settling velocity as:

$$x = \frac{\rho - \rho_0}{\rho_0} \tag{3}$$

Therefore:

$$\rho = \rho_0 x + \rho_0 = \rho(x) \tag{4}$$

If the random variable ρ has the distribution expressed by a frequency function $f(\rho)$, the frequency function of the random variable X is expressed according to the theorem of functions of random variables (Gerstenkorn and Śródka, 1972) by the following formula:

$$f_1(x) = f[\rho(x)]\left|\frac{d\rho(x)}{dx}\right| \tag{5a}$$

$$f_1(x) = f(\rho = \rho_o x + \rho_o)\rho_o \tag{5b}$$

The random variable $X^{1/2}$, according to the same theorem, will have the following distribution:

a) $Y_1 = X^{1/2}$ $\qquad x = y_1^2 = x(y_1)$

$$f_2(y_1) = f_1[x(y_1)]\,2y_1 \tag{6a}$$

$$f_2(y_1) = f_1\left(x = y_1^2\right)2y_1 \tag{6b}$$

Analogically, the random variable D occurs in Eq.(2) to 0.5 power. If $g(d)$ is the frequency function of variable D, the random variable $Y_2 = D^{1/2}$ will have the following distribution:

$$f_2(y_2) = g[d(y_2)]\,2y_2 \tag{7a}$$

$$f_2(y_2) = g\left(d = y_2^2\right)2y_2 \tag{7b}$$

$$d = y_2^2 = d(y_2) \tag{8}$$

According to the above transformations, the particle settling velocity, as the random variable *V*, will be expressed by the following formula:

$$V = 4{,}43 Y_1 Y_2 \tag{9}$$

Denoting:

$$W = 4{,}43 Y_1 \tag{10}$$

formula (9) will take the form:

$$V = W Y_2 \tag{11}$$

and the distribution of the random variable W is:

$$f_4(w) = f_2[y_1(w)]\frac{1}{4{,}43} \tag{12a}$$

$$f_4(w) = f_2\left(y_1 = \frac{w}{4{,}43}\right)\frac{1}{4{,}43} \tag{12b}$$

$$y_1(w) = \frac{w}{4{,}43} \tag{13}$$

As it can be seen from Eq.(11), settling velocity is the product of two random variables. The frequency function of the random variable, which is the product of two independent random variables $S = T\ U$, is expressed by the following formula (Gerstenkorn and Śródka, 1972):

$$h(s) = \int f_1(t) f_2\left(\frac{s}{t}\right)\frac{1}{t}dt \tag{14}$$

where: f_1 and f_2 are the frequency functions of random variables T and U, respectively. According to Eq.(14), the frequency function of settling velocity is:

$$h(v) = \int_{w_{\min}}^{w_{\max}} f_4(w)\, f_3\left(\frac{v}{w}\right)\frac{1}{w}dw \tag{15a}$$

$$h(v) = \int_{w_{\min}}^{w_{\max}} f_4(w)\, f_3\left(y_2 = \frac{v}{w}\right)\frac{1}{w}dw \tag{15b}$$

DISTRIBUTION OF SETTLING VELOCITY FOR LINEAR DISTRIBUTIONS OF PARTICLE DENSITY AND SIZE

As an example, we calculated the distribution of settling velocity for four combinations of linear frequency functions of particle size and density.

1. The sample contains mostly fine particles of low density for the ranges of particle size and density given below:

$$g(d) = (-5{,}54d + 0{,}11085)\cdot 10^3 \qquad \text{for } d \in [0{,}001; 0{,}02] \tag{16}$$

and

$$f(\rho) = -8{,}89\cdot 10^{-7}\rho + 2{,}445\cdot 10^{-3} \qquad \text{for } \rho \in [1250; 2750] \tag{17}$$

where d is expressed in [m] while ρ in [kg/m^3]. Both functions are normalized to 1,

$$\text{i.e.: } \int_{0{,}001}^{0{,}02} g(d)dd = 1 \quad \text{and} \quad \int_{1250}^{2750} f(\rho)d\rho = 1\,.$$

The cumulative distribution functions of particle size and particle density are presented in Figs 1 - 2.

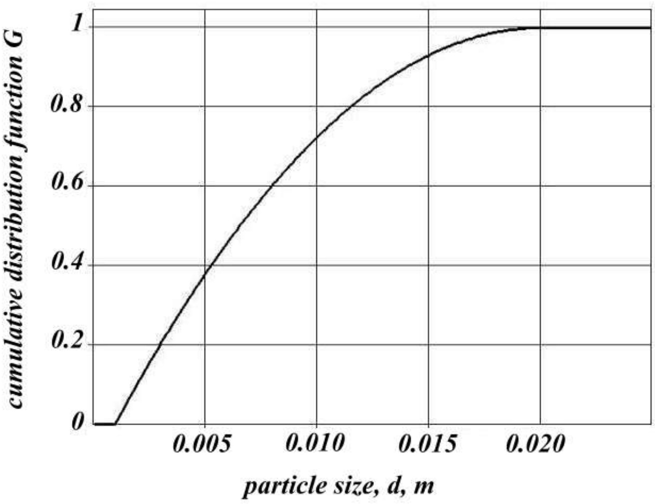

Fig. 1. Cumulative distribution function of particle size

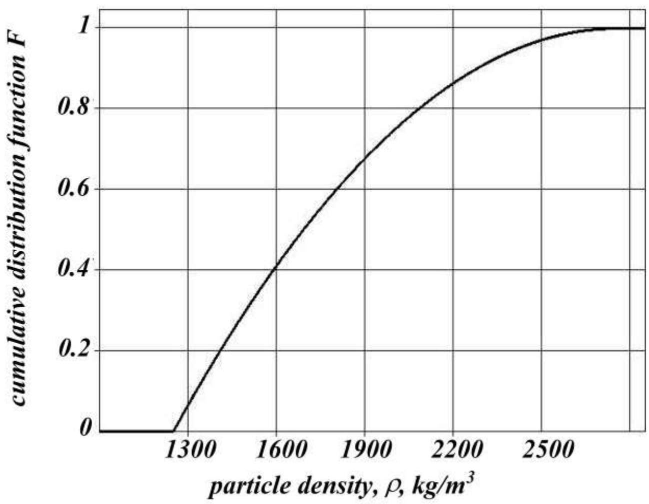

Fig. 2. Cumulative distribution function of particle density

In order to calculate the distribution of settling velocity for Newton's range (Eq.15), the distributions $f_1(x)$, $f_2(y_1)$, $f_3(y_2)$, $f_4(w)$ and $f_3\left(\frac{v}{w}\right)$ should be calculated. Function $f_1(x)$, according to Eqs (5b) and (17), is equal to:

$$f_1(x) = -0{,}889x+1{,}556 \qquad x \in [0{,}25;1{,}75] \tag{18}$$

Function $f_2(y_1)$, according to Eq.(6b), is as follows:

$$f_2(y_1) = -1{,}778y_1^3 + 3{,}112y_1 \qquad y_1 \in [0{,}5;1{,}32] \qquad (19)$$

Function $f_3(y_2)$, according to Eq.(7b), is:

$$f_3(y_2) = (-11{,}08y_2^3 + 0{,}2217y_2)\cdot 10^3 \qquad y_1 \in [0{,}03;0{,}14] \qquad (20)$$

Function $f_4(w)$, according to formula (12b), is equal to:

$$f_4(w) = -4{,}62\cdot 10^{-3}w^3 + 0{,}16w \qquad w \in [2{,}215;5{,}848] \qquad (21)$$

Substituting distributions $f_4(w)$ and $f_3\left(\frac{v}{w}\right)$ into Eq.(15b), after integration and normalization to 1, the following formulas for the frequency of settling velocity and cumulative distribution functions are obtained:

$$h(v) = -117{,}12v^3 + 22{,}23v \qquad v \in [0{,}07;0{,}44]\ [\mathrm{m/s}] \qquad (22)$$

$$H(v) = \int_{0,07}^{v} h(v)dv = -29{,}28v^4 + 11{,}113v^2 - 0{,}054 \qquad (23)$$

Figure 3 presents the cumulative distribution function of settling velocity. As it can be seen in Fig.3, the largest fraction is constituted by the particles whose settling velocity is placed in the middle of the range of obtained values.

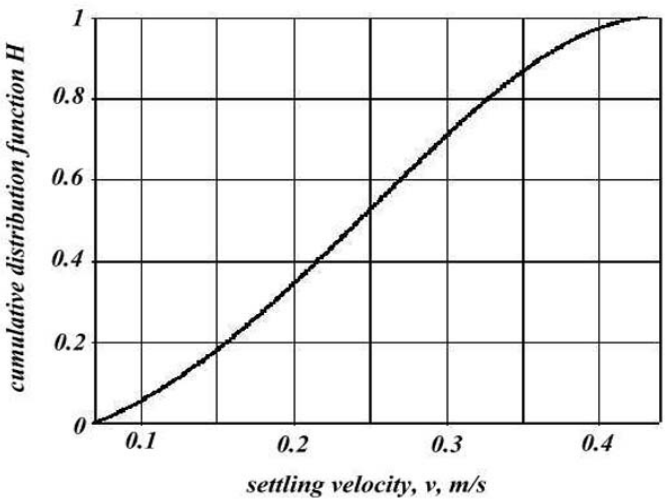

Fig. 3. Cumulative distribution function of particle settling velocity according to Eq.(23)

2. The sample contains mostly fine particles of high density. The normalized frequency functions of particle size and density are:

$$g(d) = (-5{,}54d + 0{,}11085)\cdot 10^3 \qquad \text{for } d \in [0{,}001;0{,}02] \tag{24}$$

$$f(\rho) = 8.89 \cdot 10^{-7}\rho - 1{,}11 \cdot 10^{-3} \qquad \text{for } \rho \in 1250;2750] \tag{25}$$

Their cumulative distribution functions are shown on Figs 4 - 5.

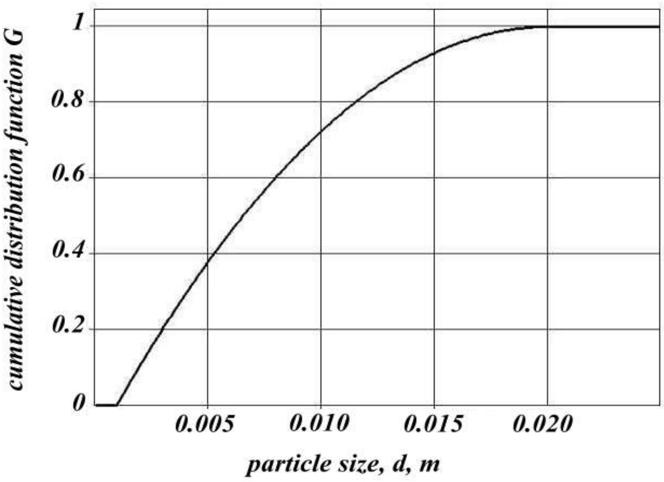

Fig. 4. Cumulative distribution function of particle size

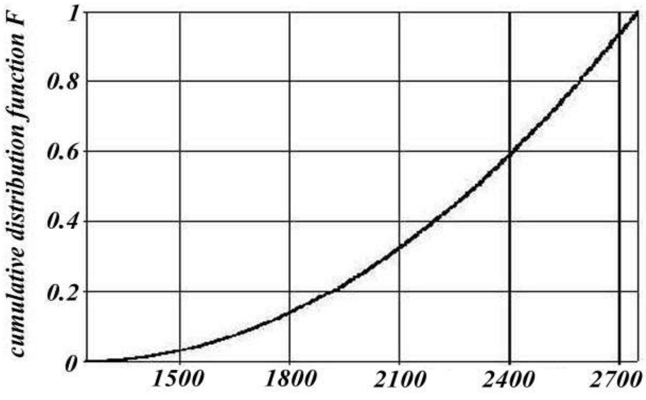

Fig. 5. Cumulative distribution function of particle density

Acting as in the section 1, the normalized frequency of settling velocity and the cumulative distribution functions are as follows:

$$h(v) = -32{,}588v^3 + 11{,}58v \qquad v \in [0{,}07; 0{,}605] \quad [\text{m/s}] \tag{26}$$

$$H(v) = -8{,}147v^4 + 5{,}79v^2 - 0{,}028 \tag{27}$$

Figure 6 shows the cumulative distribution function of settling velocity. A comparison of Figs 3 and 6 indicates that in both cases the particles with the settling velocity in the middle range of obtained velocities are prevailing. However, in the second case the value of the maximum velocity is higher.

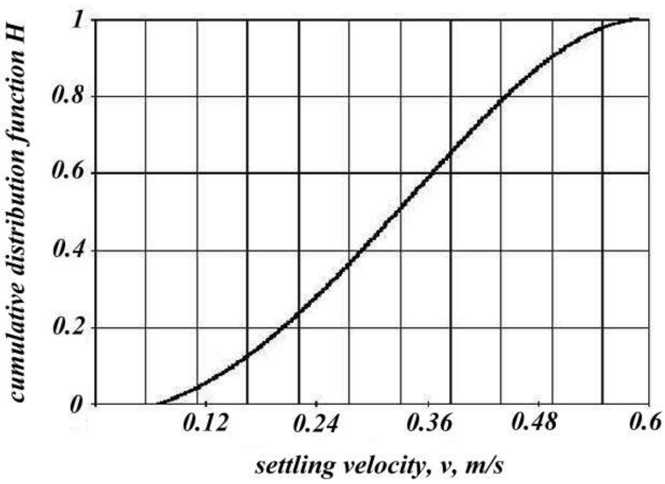

Fig. 6. The cumulative distribution function of settling velocity, according to Eq.(27)

3. The sample contains mostly large particles of low density. The normalized frequency functions can be expressed by the equations:

$$g(d) = 5{,}54 \cdot 10^3 d - 5{,}54 \qquad \text{for } d \in [0{,}001; 0{,}02] \tag{28}$$

$$f(\rho) = -8{,}89 \cdot 10^{-7} \rho + 2{,}445 \cdot 10^{-3} \qquad \text{for } \rho \in [1250; 2750] \tag{29}$$

For these distributions, and according to the above algorithm, the frequency of settling velocity and the cumulative distribution functions are given by the formulas:

$$h(v) = 99{,}137v^3 - 1{,}618v \qquad v \in [0{,}07; 0{,}467] \quad [\text{m/s}] \tag{30}$$

$$H(v) = 24{,}784v^4 - 0{,}809\,v^2 - 0{,}004 \tag{31}$$

Figure 7 presents the graph of the cumulative distribution function of settling velocity.

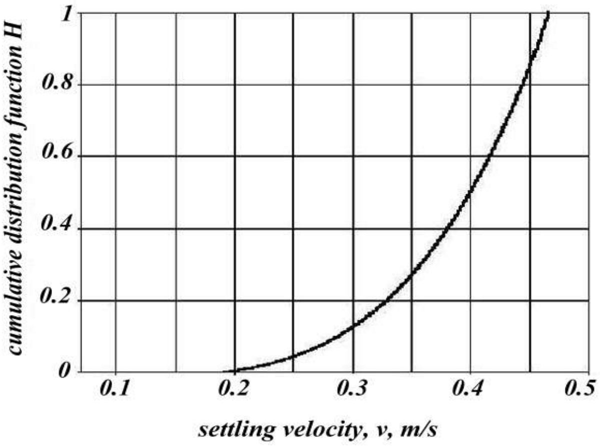

Fig. 7. Cumulative distribution function of settling velocity according to Eq.(31)

For the distributions of particle size and density given in Eqs (28)-(29) the particles of higher settling velocities dominate in the sample.

4. The sample contains mostly large particles of high densities. The frequency functions of particle size and density are given by:

$$g(d)=5{,}54\cdot 10^3 d - 5{,}54 \qquad \text{for } d \in [0{,}001;0{,}02] \tag{32}$$

$$f(\rho) = 8{,}89\cdot 10^{-7}\rho - 1{,}11\cdot 10^{-3} \qquad for\ \ \rho \in [1250;2750] \tag{33}$$

The cumulative distribution functions are presented in Figs 8-9.

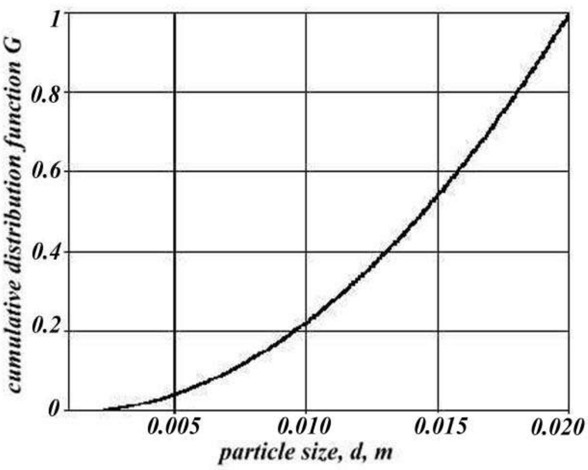

Fig. 8. Cumulative distribution function of particle size

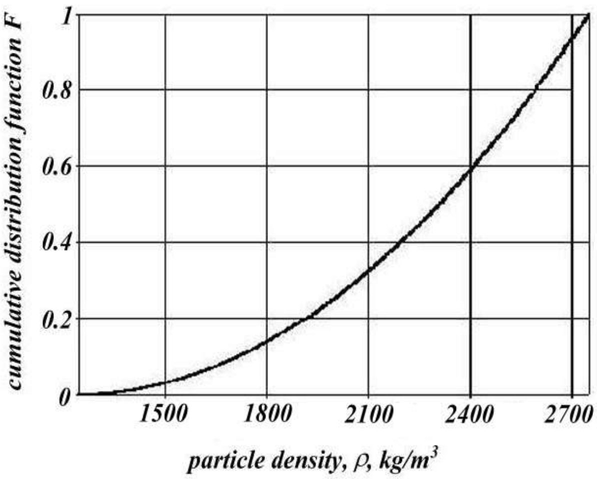

Fig. 9. Cumulative distribution function of particle density

The calculated frequency and the cumulative distribution functions of settling velocity are as follows:

$$h(v) = 26{,}563\, v^3 - 0{,}483\, v \qquad v \in [0{,}07; 0{,}636] \ \text{[m/s]} \tag{34}$$

$$H(v) = 6{,}641\, v^4 - 0{,}2415\, v^2 \tag{35}$$

Figure 10 presents the cumulative distribution function of settling velocity according to Eq.(35).

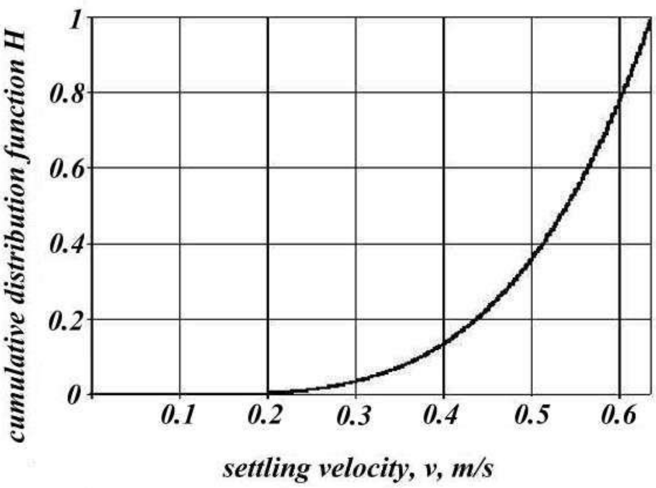

Fig. 10. The cumulative distribution function of settling velocity according to Eq.(35)

It can be seen in Fig.10, that the distributions of geometrical and physical properties, determined by Eqs (32) and (33), the particles of higher settling velocity dominate in the sample, analogically to the case considered in section 3. However, the maximum value of settling velocity is higher.

CONCLUDING

The presented in this work methods of determining the distributions of particle settling velocities are valid for random variables of stochastically independent of particle size and density. Numerous investigations of this issue prove independence of these random variables (Brożek, 1993, Tumidajski, 1997).

In order to determine the distribution of settling velocity of irregular particles it is necessary to consider their shape. The investigations indicate that the distributions of shape coefficients of coal particles are of the so-called gamma type (Brożek and Turno, 2004, Hodenberg, 1998). Also the distributions of particle size and density in case of fine coal particles (Brożek and Surowiak, 2004) are independent.

Distribution of settling velocities is the main parameter of hydraulic classification applied for fine coal separation in which the motion of particles is of turbulent character. Presented in the paper simulation of distribution of settling velocity as a function of distribution of particle size and density describes the velocity distribution changes due to sample characteristics. The distribution of settling velocity affects the separation efficiency measured by the probable error, determined by means of the densimetric analysis.

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Prędkość opadania jest argumentem rozdziału procesu wzbogacania w osadzarce. Rozkład prędkości opadania stanowi więc charakterystykę materiału kierowanego do procesu wzbogacania.

W artykule przedstawiono metodykę wyznaczania rozkładu prędkości opadania w próbce ziaren sferycznych dla turbulentnego charakteru ruchu ziaren, w którym prędkość opadania wyraża się wzorem Newtona-Rittingera. Ze względu na to, że zarówno gęstość jak i wielkość ziarna są zmiennymi losowymi o pewnych rozkładach również prędkość opadania jako funkcja tych zmiennych jest zmienną losową. Korzystając z twierdzeń rachunku prawdopodobieństwa odnoszących się do funkcji zmiennych losowych podano wzór na funkcję gęstości rozkładu prędkości opadania oraz wyliczono rozkłady prędkości dla kilku kombinacji rozkładów wielkości i gęstości ziarna.